A Real Life Application of Linear Programming

Win Win Myo[*]

Abstract

Linear programming is heavily used in microeconomics and company management, such as planning, production, transportation, technology and other issues, either to maximize the income or minimize the costs of a production scheme. In the real world the problem is to find the maximum profit for a certain production. In "real life", linear programming is part of a very important area of mathematics called "optimization techniques". In this paper, it is to be investigated two different solving graphical methods for some real life problems. Then we may introduce a new program for linear programming which is my own invention software. This system is computerized system using Microsoft Visual Basic Programming Software. This software may be helpful to solve the linear programming problems to get quickly and easily optimal solutions for any user.

Introduction

Linear programming is used to find the best or optimal solution to a problem that requires a decision or set of decisions about how best to use a set of limited resources to achieve a state goal of objectives.

Applications

Many real world problems lend themselves to linear programming modeling. Many real world problems can be approximated by linear models.

There are well-known successful applications in: manufacturing, marketing, finance (investment), advertising and agriculture.

History of Linear Programming

It started in 1947 when G. B. Dantzig design the "simplex method" for solving linear programming formulations of U.S. Air Force planning problems. It soon became clear that a surprisingly wide range of apparently unrelated problems in production management could be stated in linear programming terms and solved by the simplex method. Later, it was used to solve problems of management. Its algorithm can also be used to network flow problems. On Oct.14th,1975, the Royal Sweden Academy of Science awarded the Nobel Prize in economic science to L.V.Kantorovich and T.C.Koopmans "for their contributions to the theory of optimum allocation of resources" The breakthrough in looking for a theoretically satisfactory algorithm to solve LP problems came in 1979 when L.G.Khachian published a description of such an algorithm.

The Linear Programming Model

Let: $X_1, X_2, X_3, \ldots\ldots, X_n$ = decision variables

Z = Objective function or linear function

Requirement: Maximization of the linear function Z.

$Z = c_1X_1 + c_2X_2 + c_3X_3 + \ldots\ldots + c_nX_n$ …..Eq (1)

Subject to the following constraints:

[*] Lecturer, Department of Mathematics, Dagon University

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$
$$\vdots \qquad\qquad \vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_n$$
$$\text{all } x_j \geq 0$$

where a_{ij}, b_i, and c_j are given constants.

Developing LP Model (1)

The variety of situations to which linear programming has been applied ranges from agriculture to zinc smelting.

Steps Involved:

Determine the objective of the problem and describe it by a criterion function in terms of the decision variables.

Find out the constraints

Do the analysis which should lead to the selection of values for the decision variables that optimize the criterion function while satisfying all the constraints imposed on the problem.

Developing LP Model (2)

Example (1): Real life application problem (1) (Product Mix Problem) The N. Dustrious Company produces two products: I and II. The raw material requirements, space needed for storage, production rates, and selling prices for these products are given in Table 1.

TABLE .1 Production Data for N. Dustrious Company

	Product	
	I	II
Storage space (ft²/unit)	4	5
Raw material (lb/unit)	5	3
Production rate (units/hr)	60	30
Selling price ($/unit)	13	11

The total amount of raw material available per day for both products is 1575lb. The total storage space for all products is 1500 ft², and a maximum of 7 hours per day can be used for production.

Developing LP Model (3)

Example Problem

All products manufactured are shipped out of the storage area at the end of the day. Therefore, the two products must share the total raw material, storage space, and production time. The company wants to determine how many units of each product to produce per day to maximize its total income.

Solution

The company has decided to maximize its sale income, which depends on the number of units of product I and II. Therefore, the decision variables, x_1 and x_2 can be the number of units of products I and II, respectively, produced per day.

Developing LP Model (4)

The object is to maximize the equation:

$$Z = 13x_1 + 11x_2$$

Subject to the constraints on storage space, raw materials, and production time.

Each unit of product I requires 4 ft^2 of storage space and each unit of product II requires 5 ft^2. Thus a total of $4x_1 + 5x_2$ ft^2 of storage space is needed each day. This space must be less than or equal to the available storage space, which is 1500 ft^2. Therefore,

$4x_1 + 5x_2 \leq 1500$

Similarly, each unit of product I and II requires 5 and 3 1bs, respectively, of raw material. Hence a total of $5x_1 + 3x_2$ Ib of raw material is used.

Developing LP Model (5)

This must be less than or equal to the total amount of raw material available, which is 1575 Ib. Therefore,

$$5x_1 + 3x_2 \leq 1575$$

Prouct I can be produced at the rate of 60 units per hour. Therefore, it must take I minute or 1/60 of an hour to produce I unit. Similarly, it requires 1/30 of an hour to produce 1 unit of product II. Hence a total of $x_1/60 + x_2/30$ hours is required for the daily production. This quantity must be less than or equal to the total production time available each day. Therefore,

$$x_1/60 + x_2/30 \leq 7$$

or $x_1 + 2x_2 \leq 420$

Finally, the company cannot produce a negative quantity of any product, therefore x_1 and x_2 must each be greater than or equal to zero.

Developing LP Model (6)

The linear programming model for this example can be summarized as:

Maximize

$$Z = 13x_1 + 11x_2$$

subject to:

$$4x_1 + 5x_2 \leq 1500$$
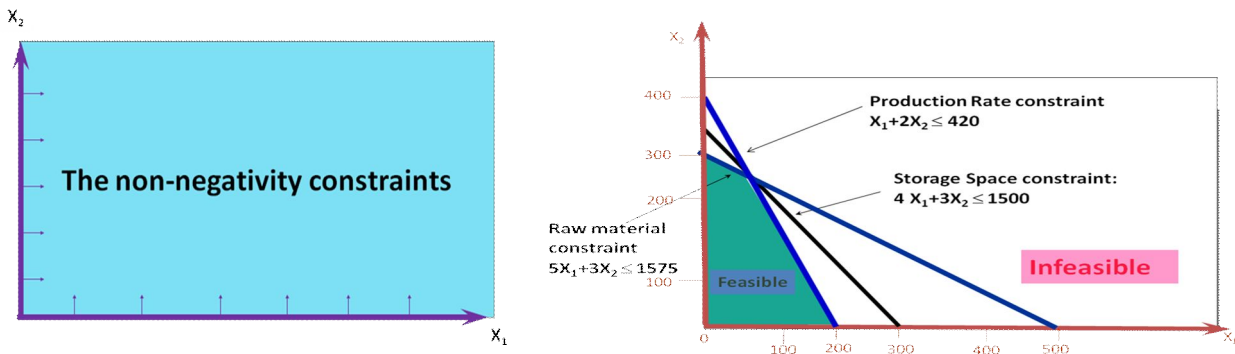$$5x_1 + 3x_2 \leq 1575$$
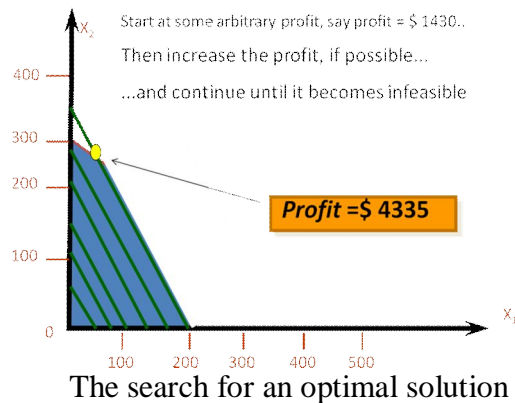$$x_1 + 2x_2 \leq 420$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

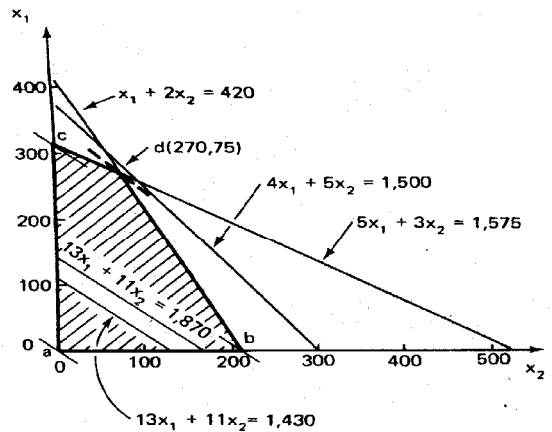## The Graphical Analysis of Linear Programming

The set of all points that satisfy all the constraints of the model is called a FEASIBLE REGION. Using a graphical presentation we can represent all the constraints, the objective function, and the three types of feasible points.

Graphical Analysis – the Feasible Region

The search for an optimal solution

Graphical Solution to LP Problem (1)

Graphical Solution to LP Problem (2)

An equation of the form $4x_1 + 5x_2 = 1500$ defines a straight line in the x_1-x_2 plane. An inequality defines an area bounded by a straight line. Therefore, the region below and including the line $4x_1 + 5x_2 = 1500$ in the Figure represents the region defined by $4x_1 + 5x_2 \leq 1500$. Same thing applies to other equations as well. The **shaded area** of the figure comprises the area common to all the regions defined by the constraints and contains all pairs of x_I and x_2 that are feasible solutions to the problem. This area is known as the feasible region or feasible solution space. **The optimal solution must lie within this region**.

There are various pairs of x_1 and x_2 that satisfy the constraints such as:

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Graphical Solution to LP Problem (3)

Trying different solutions, the optimal solution will be:

$$X_1 = 270$$

$$X_2 = 75$$

This indicates that maximum income of $4335 is obtained by producing 270 units of product I and 75 units of product II. In this solution, all the raw material and available time are used, because the optimal point lies on the two constraint lines for these resources. However, 1500- [4(270) + 5(75)], or 45 ft^2 of storage space, is not used. Thus the storage space is not a constraint on the optimal solution; that is, more products could be produced before the company ran out of storage space. Thus this constraint is said to be slack.

Graphical Solution to LP Problem (4)

If the objective function happens to be parallel to one of the edges of the feasible region, any point along this edge between the two extreme points may be an optimal solution that maximizes the objective function. When this occurs, there is no unique solution, but there is an infinite number of optimal solutions.The graphical method of solution may be extended to a case in which there are three variables. In this case, each constraint is represented by a plane in three dimensions, and the feasible region bounded by these planes is a polyhedron.

Example (2): Real life application problem (2)

	RESOURCE REQUIREMENTS		
PRODUCT	Labor (hr/unit)	Clay (lb/unit)	Revenue ($/unit)
Bowl	1	4	40
Mug	2	3	50

There are 40 hours of labor and 120 pounds of clay available each day

Decision variables
x_1 = number of bowls to produce
x_2 = number of mugs to produce

LP Formulation: Example

Maximize $Z = \$40\, x_1 + 50\, x_2$

Subject to

$$x_1 + 2x_2 \quad £\ 40 \text{ hr (labor constraint)}$$

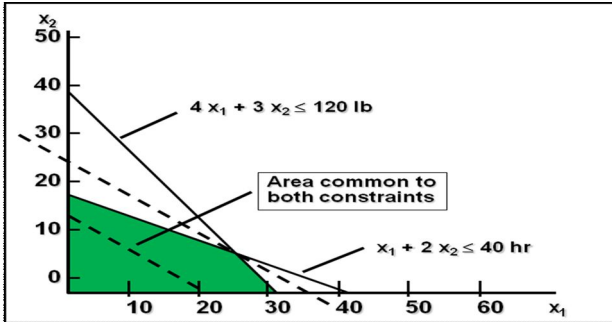$$4x_1 + 3x_2 \quad £\ 120 \text{ lb} \qquad \text{(clay constraint)}$$

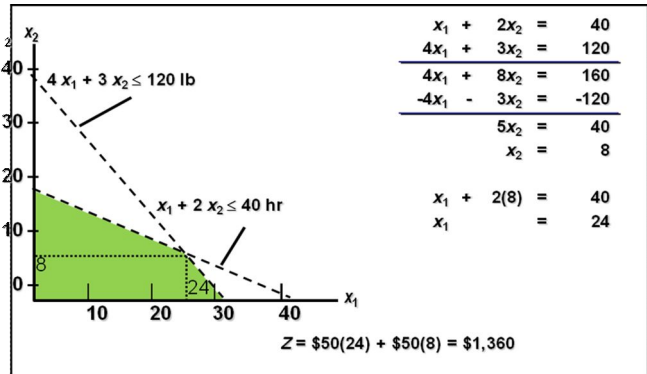$$x_1, x_2 \quad ³\ 0$$

Solution is $x_1 = 24$ bowls $x_2 = 8$ mugs

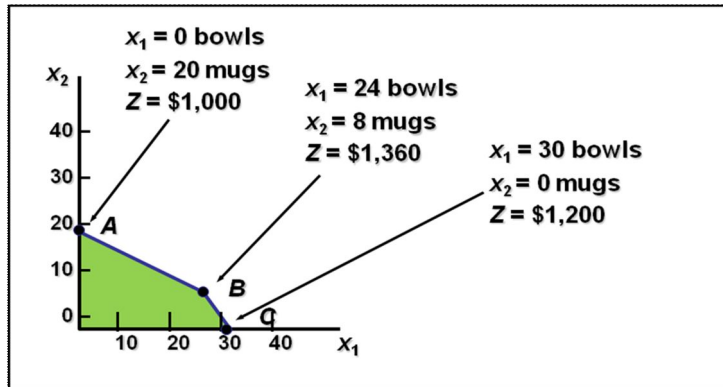Revenue = $1,360

Graphical Solution Method

Plot model constraint on a set of coordinates in a plane. Identify the feasible solution space on the graph where all constraints are satisfied simultaneously. Plot objective function to find the point on boundary of this space that maximizes (or minimizes) value of objective function.

Graphical Solution (as an example)

Computing Optimal Values

Extreme Corner Points

Linear Programming Software

proLP. exe

Created by
Win Win Myo

Using proLP Software to find an Optimal Solution

To see the input screen Double click proLP icon.

How to use proLP Software

- ❖ To input the data click entry data button
- ❖ To obtain the optimal solution click solve button
- ❖ To entry the new data or to delete the old data click cancel button
- ❖ To save the problem and solution click the save button
- ❖ To print the problem and solution click the print button

Conclusion

Linear programming has proven to be an extremely powerful tool, both in modering real-word problems and as a widely applicable mathematical theory. Linear programming theory falls within convex optimization theory and is also considered to be an important part of business and economics, but may also be used to solve certain engineering problems.

Acknowledgements

References

Strayer, J. K. (1989). "Linear Programming and Applicatioons"

Vasek Chv et al. (1983). "Linear Programming"